Some Numerical Excercises on Hypothesis

Q1. Tata Motor supplies a rear axle for Postal service mail trucks. These axles must be able to withstand 80000 pounds per square inch in stress tests, but an excessively strong axle raises production costs significantly. Long experience indicates that the standard deviation of the strength of its axles is 4000 pounds per square inch. The Manufacturer selects a sample of 100 axles from production, tests them, and finds that the mean stress capacity of the sample is 79600 pounds per square inch.

i. Formulate the null and alternate hypothesis

ii. If manufacturer uses a significance level of 0.05 in testing, will the axles meet his stress requirements? (Take Z = 1.96) Ans. Null hypothesis is accepted

Step 1: H_0: μ = 80000 Pounds per square inch

Step 2: H_1: $\mu \neq$ 80000 Pounds per square inch

Step 3: α = 0.05

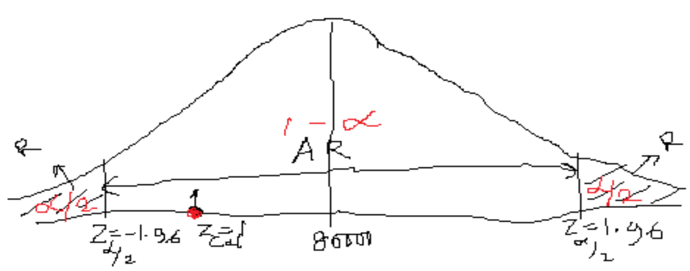
Step 4: $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ **(since sample size is >=30 and population SD is given, so Z distribution is applicable)**

Step 5: Sample size, n = 100; $\bar{X} = 79600$; σ = **4000; μ =80000**

$$Calcualted \left| Z \right| = \left| \frac{79600 - 80000}{\frac{4000}{\sqrt{100}}} \right| = \left| -1 \right| = 1$$

Step 6: Critical value (Table value at 5% (.05) significance level for two tail test is) +/- 1.96

Step 7: Since, calculated value of |Z| score is **less than the critical value** (Table value at 5% (.05) significance level and falls in acceptance zone, null hypothesis is accepted. Hence we conclude that manufacturer's claim that rear axle stress requirement of 80000 per square inch is accepted at 5% level of significance



Q2. SBI claims that more than 55% of the saving accounts in Patna are at SBI. A sample survey of 400 account holders revealed that only 180 account holders have account at SBI.

i. Formulate the null and alternate hypothesis

ii. Verify, using 5% level of significance, if the sample results underestimate the claim of SBI.

Step 1: H_0: P > 0.55 (It is hypothesised that more than 55% account holders in Patna are SBI customer)

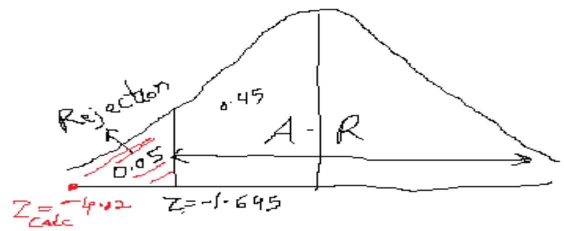
Step 2: H_1: P <= 0.55

Step 3: α = 0.05

Step 4: $z = \frac{\bar{p}-p}{\sigma_{\bar{p}}}$ **(since sample size is > 30, so Z distribution is applicable). Here population is infinite.**

Step 5: Sample size, n = 400; $\overline{P} = \frac{180}{400} = 0.45$; **P = 0.55;** $\sigma_{\overline{p}} = \sqrt{\frac{p\times(1-p)}{n}} = \sqrt{\frac{0.55\times(0.45)}{400}} = 0.024875$

$$Calcualted\ Z = \frac{0.45 - 0.55}{.024875} = -4.02$$



Step 6: Critical value (Table value at 5% (.05) significance level for one and left tail test) is – 1.645

Step 7: Since, calculated value of │Z score is **less than the critical value** (Table value at 5% (.05) significance level and falls in rejection zone, null hypothesis is not accepted.

Hence we conclude that hypothesis of SBI claims that more than 55% of the saving accounts in Patna are at SBI is not accepted at 5% level of significance

Q3. A certain chemical process is said to have produced 15 or less pounds of waste material for every 60 lbs. batch with a corresponding standard deviation of 5 lbs. A random sample of 100 batches gives an average of 16 lbs. of waste per batch. Test at 10 per cent level whether the average quantity of waste per batch has increased. Compute the power of the test for μ = 16 lbs. If we raise the level of significance to 20 per cent, then how the power of the test for μ = 16 lbs. would be affected?

Ans: reject H_0 *at 10%, power of test (1-β) is* 0.7642; power of test at 20% is .8770

Step 1: H_0: μ <= 15

Step 2: H_1: μ > 15

Step 3: α = 0.1

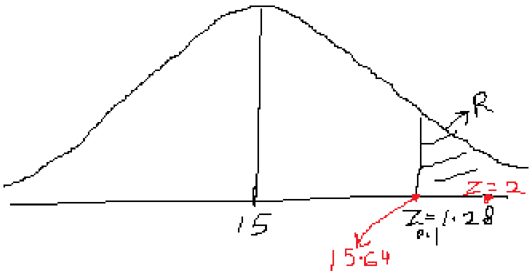
Step 4: $z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ **(since sample size is > 30 and population SD is given, so Z distribution is applicable).**

Step 5: Sample size, n = 100; $\bar{X} = 16$; σ **= 5; μ =15**

$$Calcualted\ Z = \frac{16 - 15}{5/\sqrt{100}} = 2$$

Step 6: Critical value (Table value at 10% (.1) significance level for one tail test is) + 1.28

Step 7: Since, calculated value of │Z│ score is **more than the critical value** (Table value at 10% (.1) significance level and falls in rejection zone, null hypothesis is not accepted. Hence we conclude that the average quantity of waste per batch has increased significantly at 10% level of significance.



Accept H_0 if $\bar{x} + 1.28\frac{\sigma}{\sqrt{n}} = 15 + 1.28 \times \frac{5}{\sqrt{100}} = 15.64$

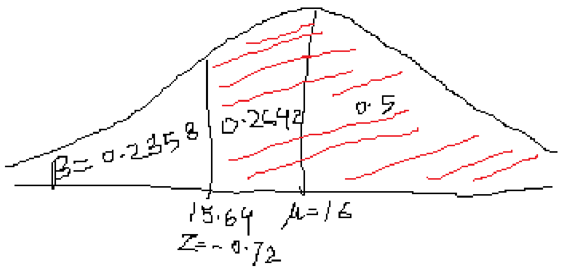
$\beta = P(\text{Accept } H_0 : \bar{x} \leq 15.64 \mid \mu = 16)$

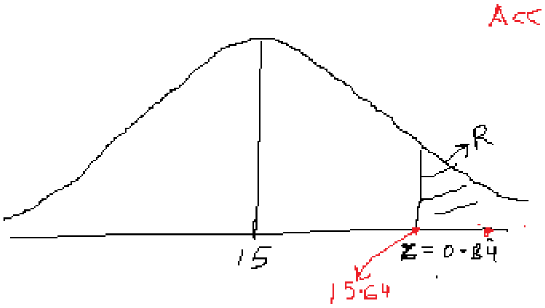
$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{15.64-16}{5/\sqrt{100}} = -0.72$

Prob at $z = -0.72$ is 0.2642

$\beta = 0.5 - 0.2642 = 0.2358$

Power of test $= 1-\beta = 1 - 0.2358 = 0.7642$





Accept H_0 if $\bar{x} + 0.84\frac{\sigma}{\sqrt{n}} = 15 + 0.84 \times \frac{5}{\sqrt{100}} = 15.42$

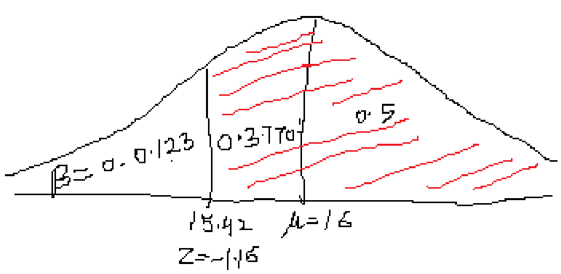
$\beta = P(\text{Accept } H_0 : \bar{x} \leq 15.42 \mid \mu = 16)$

$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{15.42-16}{5/\sqrt{100}} = -1.16$

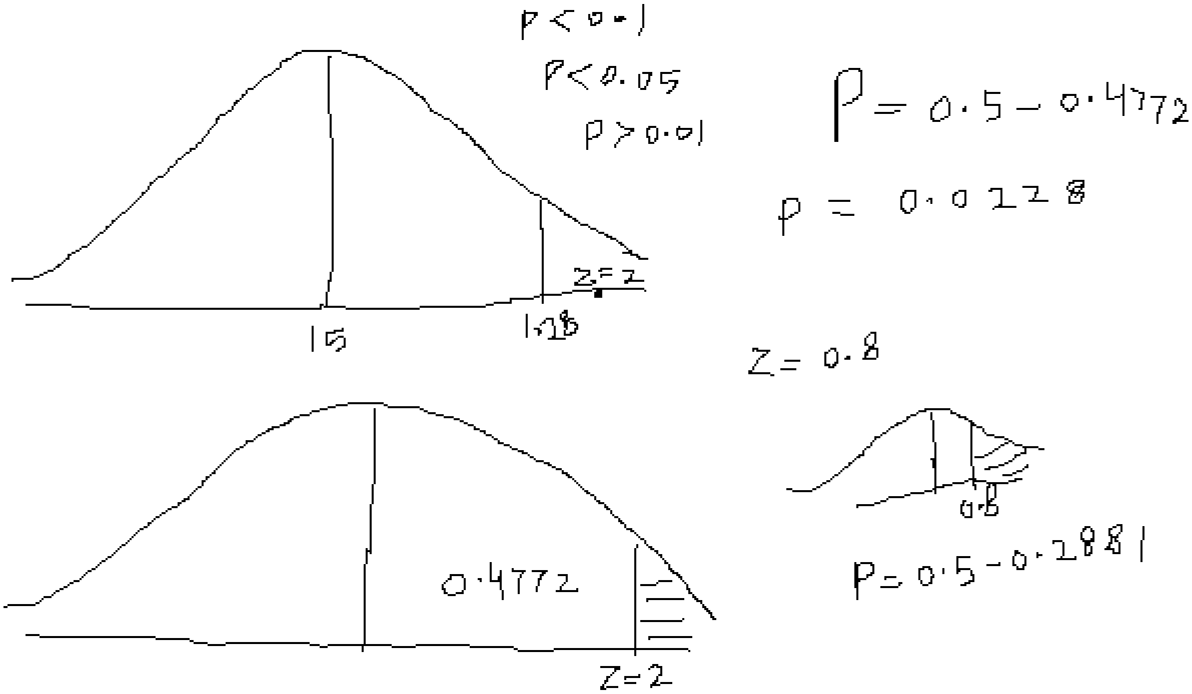
Prob at $z = -1.16$ is 0.377

$\beta = 0.5 - 0.377 = 0.123$

Power of test $= 1-\beta = 1 - 0.123 = 0.877$



P Value



Q4. A sample of 400 male students is found to have a mean height 67.47 inches. Can it be reasonably regarded as a sample from a large population with mean height 67.39 inches and standard deviation 1.30 inches? Test at 5% level of significance. Ans. Null hypothesis is accepted.

Q5. Suppose we are interested in a population of 20 industrial units of the same size, all of which are experiencing excessive labour turnover problems. The past records show that the mean of the distribution of annual turnover is 320 employees, with a **standard deviation** of 75 employees. A sample of 5 of these industrial units is taken at random which gives a mean of annual turnover as 300 employees. Is the sample mean consistent with the population mean? Test at 5% level. Ans. Null hypothesis is accepted.

Step 1: H_0: $\mu = 320$

Step 2: H_1: $\mu \neq 320$

Step 3: $\alpha = 0.05$

Step 4: $z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}}$ **(since sample size is < 30 and population SD is given, so Z distribution is applicable). Here population is finite, so population correction factor is used**

Step 5: **Population size, N = 20;** Sample size, n = 5; $\bar{X} = 300$; $\sigma = 75$; $\mu = 320$

$$Calcualted\ Z = \frac{300-320}{75/\sqrt{5}} \times \sqrt{\frac{20-5}{20-1}} = -0.67$$

Step 6: Critical value (Table value at 5% (.05) significance level for two tail test is) +/- 1.96

Step 7: Since, calculated value of $|Z|$ score is **more than the critical value** (Table value at 5% (.05) significance level and falls in acceptance zone, null hypothesis is accepted. Hence we conclude that hypothesis of sample mean employee turnover is consistent with population mean is accepted at 5% level of significance

Q6. The mean of a certain production process is known to be 50 with a standard deviation of 2.5. The production manager may welcome any change in mean value towards higher side but would like to safeguard against decreasing values of mean. He takes a sample of 12 items that gives a mean value of 48.5. What inference should the manager take for the production process on the basis of sample results? Use 5 per cent level of significance for the purpose. Ans. Null hypothesis is rejected.

Step 1: H_0: $\mu >= 50$ (It is hypothesised that mean production process value is more than or equal to 50)

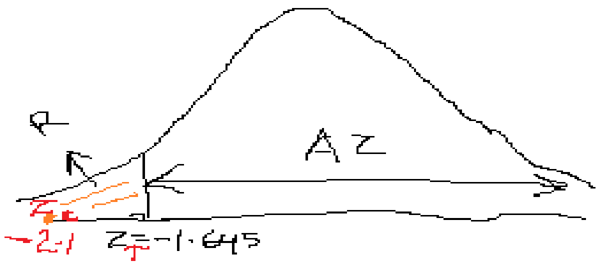
Step 2: H_1: $\mu < 50$ (It is hypothesised that mean production process value is less than 50)

Step 3: $\alpha = 0.05$

Step 4: $z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ **(for infinite population)**

Step 5: Sample size, n = 12; $\bar{X} = 48.5$; $\sigma = 2.5$; $\mu = 50$

$$z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{48.5-50}{2.5/\sqrt{12}} = -2.1$$



Step 6: Critical value (Table value at 5% (.05) significance level for two tail test is) – 1.645

Step 7: Since, calculated value of |Z| score is **less than the critical value** (Table value at 5% (.05) significance level and falls in rejection zone, null hypothesis is not accepted.

Q7. The specimens of copper wires drawn form a large lot have the following breaking strength (in kg. weight): 578, 572, 570, 568, 572, 578, 570, 572, 596, 544

Test (using Student's *t*-statistic) whether the mean breaking strength of the lot may be taken to be 578 kg. weight (Test at 5 per cent level of significance). Ans. Null hypothesis is accepted.

Step 1: H_0: $\mu = 578$

Step 2: H_1: $\mu \neq 578$

Step 3: $\alpha = 0.05$

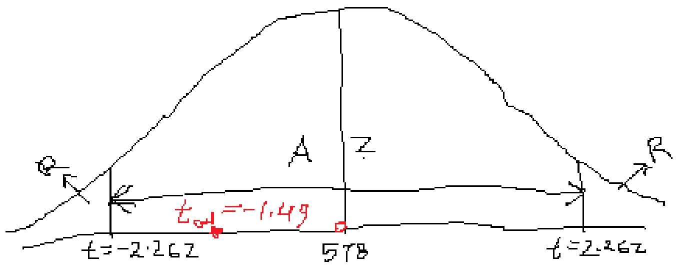
Step 4: $t = \frac{\bar{X}-\mu}{s/\sqrt{n}}$ (Since sample size is less than 30 and population SD is not given, so student t distribution will be applicable)

Step 5: Sample size, n = 10; $\bar{X} = 572$; $s = 12.72$; $\mu = 578$

$$t = \frac{\bar{X}-\mu}{s/\sqrt{n}} = \frac{572-578}{12.72/\sqrt{10}} = -1.49$$

Step 6: Critical value (Table value of student t distributionat 5% (.05) significance level and at 9 degree of freedom for two tail test is) +/-2.262

Step 7: Since, calculated value of |t| score falls in acceptance zone, null hypothesis is accepted. Hence, we conclude that the mean breaking strength of the lot may be taken to be 578 kg is true.



Q8. Raju Restaurant near the railway station at Falna has been having average sales of 500 tea cups per day. Because of the development of bus stand nearby, it expects to increase its sales. During the first 12 days after the start of the bus stand, the daily sales were as under: 550, 570, 490, 615, 505, 580, 570, 460, 600, 580, 530, 526

On the basis of this sample information, can one conclude that Raju Restaurant's sales have increased? Use 5 per cent level of significance. Ans. t = 3.558; Null hypothesis is rejected

Q9. The mean produce of wheat of a sample of 100 fields in 200 lbs. per acre with a standard deviation of **10** lbs. Another samples of 150 fields gives the mean of 220 lbs. with a standard deviation of **12** lbs. Can the two samples be considered to have been taken from the same population whose standard deviation is **11** lbs? Use 5 per cent level of significance. Ans. Null hypothesis is rejected. This means that the difference between means of two samples is statistically significant and not due to sampling fluctuations.

Solution:

Step 1: H_0: $\mu_1 = \mu_2$; or $\mu_1 - \mu_2 = 0$

Step 2: H_1: $\mu_1 \neq \mu_2$; or $\mu_1 - \mu_2 \neq 0$

Step 3: $\alpha = 0.05$

Step 4: $Z = \frac{\bar{X}_1 - \bar{X}_2}{\sigma_{\bar{X}_1 - \bar{X}_2}}$

$$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\sigma_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{11^2\left(\frac{1}{100} + \frac{1}{150}\right)} = 1.417$$

Step 5: Sample size, $n_1 = 100$; $\bar{X}_1 = 200$; $n_2 = 150$; $\bar{X}_2 = 220$; $\sigma_p = 11$

$$Calcualted\ Z = \frac{200 - 220}{1.417} = -14.11$$

Step 6: Critical value (Table value at 5% (.05) significance level for two tail test is) +/- 1.96

Step 7: Since, calculated value of |Z| score is **more than the critical value** (Table value at 5% (.05) significance level and falls in rejection zone, null hypothesis is not accepted. Hence we conclude that there is significant difference in their mean value, so they are not from same population.

Q10. A simple random sampling survey in respect of monthly earnings of semi-skilled workers in two cities gives the following statistical information:

City	Mean monthly earnings (Rs)	Standard deviation of data of sample monthly earnings(Rs.)	Size of sample
A	695	40	200
B	710	60	175

Test the hypothesis at 5 per cent level that there is no difference between monthly earnings of workers in the two cities. Ans. Reject *H*0 at 5 per cent level and conclude that earning of workers in the two cities differ significantly.

Step 1: H_0: $\mu_1 = \mu_2$; or $\mu_1 - \mu_2 = 0$

Step 2: H_1: $\mu_1 \neq \mu_2$; or $\mu_1 - \mu_2 \neq 0$

Step 3: $\alpha = 0.05$

Step 4: $Z = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_{\bar{x}_1}-\mu_{\bar{x}_2})}{\sigma_{\bar{x}_1-\bar{x}_2}} = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sigma_{\bar{x}_1-\bar{x}_2}} = \frac{(\bar{x}_1-\bar{x}_2)}{\sqrt{\frac{\sigma_{p1}^2}{n_1}+\frac{\sigma_{p2}^2}{n_2}}} = \frac{(695-710)}{\sqrt{\frac{40^2}{200}+\frac{60^2}{175}}}$ =-2.809

Q11. Sample of sales in similar shops in two towns are taken for a new product with the following results:

City	Mean sales (Rs)	Variance (Rs.)	Size of sample
A	57	5.3	5
B	61	4.8	7

Is there any evidence of difference in sales in the two towns? Use 5% level of significance for testing of difference between the means of two samples.

Step 1: H_0: $\mu_1 = \mu_2$; or $\mu_1 - \mu_2 = 0$

Step 2: H_1: $\mu_1 \neq \mu_2$; or $\mu_1 - \mu_2 \neq 0$

Step 3: $\alpha = 0.05$

Step 4: $t = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_{\bar{x}_1}-\mu_{\bar{x}_2})}{\sigma_{\bar{x}_1-\bar{x}_2}} = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sigma_{\bar{x}_1-\bar{x}_2}} = \frac{(\bar{x}_1-\bar{x}_2)}{\sqrt{\frac{(n_1-1)\sigma_{s_1}^2+(n_2-1)\sigma_{s_2}^2}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

With degree of freedom = $n_1 + n_2 - 2 = (5 + 7 - 2) = 10$

Step 5: Sample size, $n_1 = 5$; $\bar{X}_1 = 57$; $n_2 = 7$; $\bar{X}_2 = 61$; $\sigma_{s_1}^2 = 5.3$; $\sigma_{s_2}^2 = 4.8$

$$Calcualted\ t = \frac{57-61}{\sqrt{\frac{(5-1)5.3+(7-1)4.8}{5+7-2}}\sqrt{\frac{1}{5}+\frac{1}{7}}} = -3.053$$

Step 6: Critical value (Table value t at 5% (.05) significance level for two tail test at 10 dof. Is) +/- 2.228

Step 7: Since, calculated value of |t| score, 3.053 is **greater than the critical value** (Table value of t 2.228 at 5% (.05) significance level and falls in rejection zone, null hypothesis is not accepted. Hence we conclude that the difference in sales in two towns is significant at 5% level of significance.

Q11. An automatic bottling machine fills oil into *2-liter* (2,000 *cm3*) bottles. A consumer advocate wants to test the null hypothesis that the average amount filled by the machine into a bottle is at least 2,000 *cm3*. A random sample of 40 bottles coming out of the machine was selected and the exact contents of the selected bottles are recorded. The sample mean was 1,999.6 *cm3*. The population standard deviation is known from past experience to be 1.30 *cm3*.

(a) Test the null hypothesis at an α of 5%.

(b) Assume that the population is normally distributed with the same standard deviation of 1.30 *cm3*. Assume that the sample size is only 20 but the sample mean is the same 1,999.6 *cm3*. Conduct the test once again at an α of 5%.

(c) If there is a difference in the two test results, explain the reason for the difference.

Q12. An automobile manufacturer substitutes a different engine in cars that were known to have an average miles-per-gallon rating of 31.5 on the highway. The manufacturer wants to test whether the new engine changes the miles-per-gallon rating of the automobile model. A random sample of 100 trial runs gives $\bar{X}$ = 29.8 miles per gallon and S = 6.6 miles per gallon. Using the 0.05 level of significance, is the average miles-per-gallon rating on the highway for cars using the new engine different from the rating for cars using the old engine?

Q13. Sixteen oil tins are taken at random from an automatic filling machine. The mean weight of the tins is 14.2 kg, with a standard deviation of 0.40 kg. Can we conclude that the filling machine is wasting oil by filling more than the intended weight of 14 kg, at a significance level of 5%?

Q14. SBI claims that more than 55% of the saving accounts in Patna are at SBI. A sample survey of 400 account holders revealed that only 180 account holders have account at SBI. Verify, using 5% level of significance, if the sample results underestimate the claim of SBI.

Q15. A company is engaged in the packaging of a superior quality tea in jars of 500gm each. The company is of the view that as long as the jars contains 500gm of tea, the process is under control. The standard deviation of the process is 50gm. A sample of 225 jars is taken at random and the sample average is found to be 510 gm. Has the process gone out of control?

Q16. A sample of size 400 was drawn and the sample mean found to be 99. Test, at 5% level of significance, whether this sample could have come form normal population with mean 100 and variance 64.

Q17. A manufacturer of a new motorcycle claims for it an average mileage of 60 km/liter under city conditions. However, the average mileage in 16 trials is found to be 57 km, with a standard deviation of 2 km. Is the manufacturer's claim justified?

Q18. A stock-broker claims that she can predict with 85% accuracy whether a stock's market value will rise or fall during the coming month. Test the stock-broker's claim at 5% level of significance if, as a test, she predict the outcome of 6 stocks and is correct in 5 of the predictions.

Step 1: H_0: P = 0.85 (It is hypothesised that mean proportion of stock prediction accuracy is 0.85)

Step 2: H_1: P ≠ 0.85

Step 3: α = 0.05

Step 4: $t = \frac{\bar{p}-p}{\sigma_{\bar{p}}}$ **(since sample size is < 30, so t distribution is applicable). Here population is infinite.**

Step 5: Sample size, n = 6; sample proportion $\bar{P} = 0.833$; **P = 0.85;** $\sigma_{\bar{p}} = \sqrt{\frac{p\times(1-p)}{n}} = \sqrt{\frac{0.85\times(0.15)}{6}} = 0.1458$

$$Calcualted\ t = \frac{0.833 - 0.85}{.1458} = -1.116$$

Step 6: Critical value (Table value at 5% (.05) significance level for two tail test) is +/-2.571

Step 7: Since, calculated value of $|Z|$ score is **less than the critical value** (Table value at 5% (.05) significance level and falls in acceptance zone, null hypothesis is accepted.

Hence we conclude that stock broker's claim of 85% accuracy is correct at 5% level of significance

Q19. Suppose a hospital uses large quantities of packaged doses of particular drugs. The individual dose of this drug is 100 cc. The action of the drug is such that the body will harmlessly pass off excessive doses. On the other hand, insufficient doses do not produce the desired medical effect, and they interfere with patient treatment. The hospital has purchased this drug from the same manufacturer for a number of years and knows that the population S.D. is 2 cc. The hospital inspects 50 doses of this drug at random from a very large shipment and finds the mean of these dosages to be 99.75 cc. If hospital sets a 0.10 significance level and ask us whether the dosages in this shipment are two small, how can we find the answer?

Step 1: H_0: μ >= 100 (It is hypothesised that mean doses of drug is more than or equal to 100)

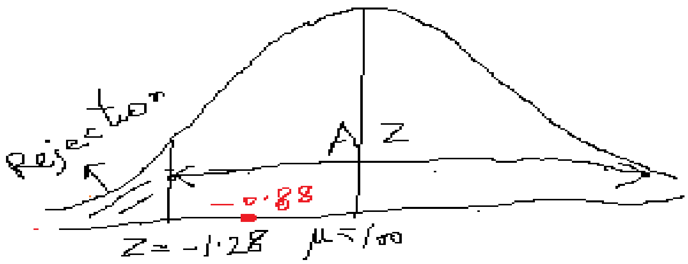
Step 2: H_1: μ < 100 (It is hypothesised that mean doses of drug is less than 100)

Step 3: α = 0.10

Step 4: $z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ **(since sample size is more than 30 and population SD is given, so z distribution is applicable. And also here population is infinite)**

Step 5: Sample size, **n** = **50**; $\bar{X} = 99.75$; σ = **2; μ =100**

$z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{99.75-100}{2/\sqrt{50}} =$ **- 0.88**

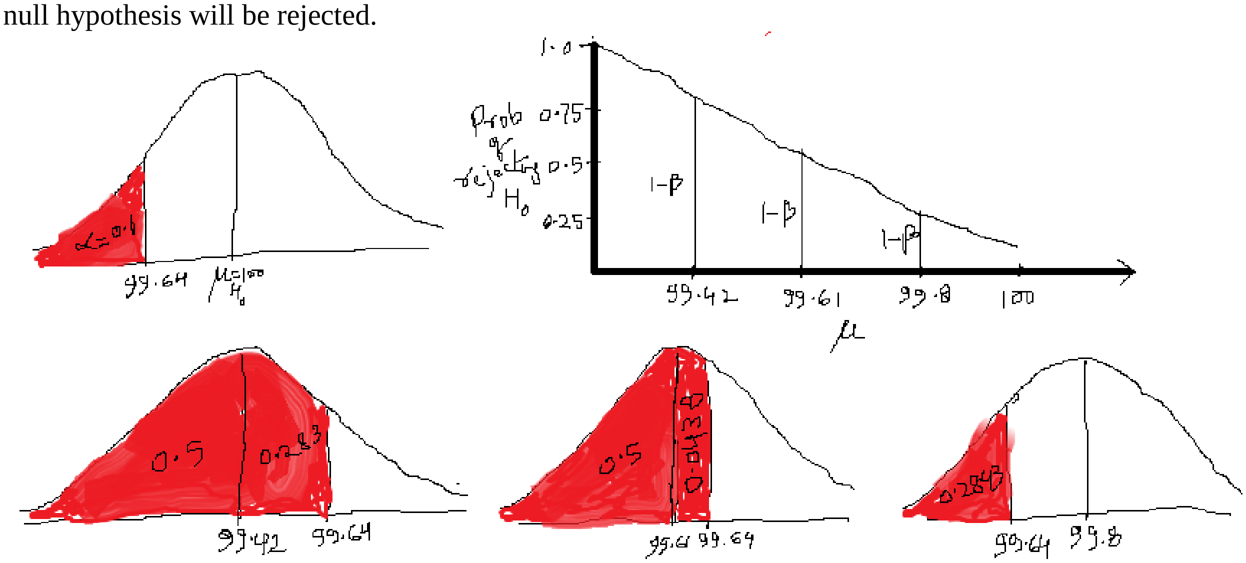


Step 6: Critical value (Table value at 10% (.10) significance level for one tail test is) – 1.645

Step 7: Since, calculated value of $|Z|$ score 0.88 is **less than the critical value** (Table value at 10% (.10) significance level and falls in aceptance zone, null hypothesis is accepted and hence we colclude that doses of drugs in the shipment are sufficient.

Measuring the power of a Hypothesis Test

If the sample mean dosage is less than$(\mu - Z_{0.10}\frac{\sigma}{\sqrt{n}}) = 100 - 1.28 \times (0.2829) = 99.64\ cc$, then null hypothesis will be rejected.



Let us consider population mean doses is 99.42 cc. then $\boldsymbol{z} = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{99.64-99.42}{2/\sqrt{50}} = \mathbf{0.78.}$

Let us consider population mean doses is 99.61 cc. then $\boldsymbol{z} = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{99.64-99.61}{2/\sqrt{50}} = \mathbf{0.11}$

Let us consider population mean doses is 99.8 cc. then $\boldsymbol{z} = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{99.64-99.8}{2/\sqrt{50}} = \mathbf{-0.565}$

P value: